the phenomenon is a consequence of the wave mechanical scattering by the region of the helium nucleus where the forces differ from inverse square forces. The present paper shows that the collisions of α-particles with helium nuclei can only be explained on the wave mechanics and it indicates also how far such calculations can give an accurate picture of the collision processes.

It is a pleasure to express my thanks to Lord Rutherford for his interest in this research and to Dr. J. Chadwick, who suggested the problem, for much valuable advice. I also wish to acknowledge my indebtedness to Mr. J. P. Gott who collaborated with me in the earlier stages of the work, and to Mr. H. M. Taylor for many helpful discussions.

The Scattering of Fast β-Particles by Electrons.

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[Plate 22.]

1. *Introduction.*

In a recent paper,† it has been shown, using the expansion method, that the simple relativistic expressions govern the transfer of momentum and energy during the close collisions of fast β-particles with electrons. The present paper gives an account of an investigation of the scattering of fast β-particles by electrons, using the expansion method.

Among the many formulæ‡ which have been proposed to express the interaction of two electrons, the relativistically invariant expression due to Möller§ appears to be the most satisfactory theoretically.|| Möller has referred the scattering for all velocities¶ to a Lorentz frame of co-ordinates in which the

† Champion, 'Proc. Roy. Soc.,' A, vol. 136, p. 630 (1932).

‡ Gaunt, 'Proc. Roy. Soc.,' A, vol. 122, p. 513 (1929); 'Phil. Trans.,' A, vol. 228, p. 151 (1929); Breit, 'Phys. Rev.,' vol. 34, p. 553 (1930); Wolfe, 'Phys. Rev.,' vol. 37, p. 591 (1931); Inglis, 'Phys. Rev.,' vol. 37, p. 795 (1931).

§ 'Z. Physik,' vol. 70, p. 786 (1931).

|| Dirac, 'Proc. Roy. Soc.,' A, vol. 135, p. 453 (1932); Heisenberg, 'Ann. Physik,' vol. 13, p. 430 (1932).

¶ The writer is indebted to Dr. Möller for communicating these results by letter.

momenta of the two electrons are equal and opposite. The observed angle of scattering θ is connected with θ*, the angle of scattering in the Lorentz frame, by the relation

$$x = \cos\theta^* = \frac{2 - (\gamma + 3)\sin^2\theta}{2 + (\gamma - 1)\sin^2\theta}, \tag{1}$$

where $\gamma = 1/(1 - \beta^2)^{\frac{1}{2}}$ and $\beta = v/c$, v being the relative velocity of the two particles before collision and c the velocity of light.

The expression for the scattering is

$$dQ(\theta) = 4\pi\left(\frac{e^2}{m_0 v^2}\right)^2 \frac{(\gamma+1)}{\gamma^2} dx \left\{\frac{4}{(1-x^2)^2} - \frac{3}{(1-x^2)} + \frac{(\gamma-1)^2}{4\gamma^2}\left[1 + \frac{4}{(1-x^2)}\right]\right\}. \tag{2}$$

It is interesting to note that the formula contains no terms in Planck's constant h. At low velocities, equation (2) becomes

$$dQ(\theta) = \left(\frac{e^2}{m_0 v^2}\right)^2 \sin 2\theta\, 2d\theta\, d\phi \left\{(\mathrm{cosec}^4\theta + \sec^4\theta - \mathrm{cosec}^2\theta \sec^2\theta) - \frac{\beta^2}{4}[4\,\mathrm{cosec}^4\theta + 3\sec^4\theta - 2\,\mathrm{cosec}^2\theta\sec^2\theta - 3\sec^2\theta]\right\}. \tag{3}$$

Now Mott† has obtained the following formula for the scattering of slow β-particles‡ by electrons :—

$$dQ(\theta) = \left(\frac{e^2}{m_0 v^2}\right)^2 \{\mathrm{cosec}^4\theta + \sec^4\theta - \mathrm{cosec}^2\theta\sec^2\theta\cos U\} \sin 2\theta\, 2d\theta\, d\phi \tag{4}$$

where

$$U = \frac{2}{137} \cdot \frac{c}{v} \cdot \log\cot\theta.$$

It is clear that the first two terms in the first bracket of (3) and (4) represent the β-particles scattered and the electrons projected through θ respectively according to classical non-relativistic theory.§ Further, when U is nearly

† 'Proc. Roy. Soc.,' A, vol. 126, p. 259 (1930).

‡ This formula has been confirmed experimentally for slow β-particles by Williams, 'Proc. Roy. Soc.,' A, vol. 128, p. 459 (1930), using the expansion method.

§ The term "classical non-relativistic theory" is used here and throughout this paper to refer to the case where the mass of the moving particles is assumed to be invariant and equal to the rest mass of the electron. Such a case does not occur in reality but we wish to compare the scattering laws in the regions where the relativity effects are appreciable, with those obtained by a complete neglect of relativity. Further, we wish to see what empirical changes we must make in a completely non-relativistic formula in order that the new formula may agree with experiment. The rest mass, in this paper, is always denoted by m_0, but it should be observed that in much of the existing literature, especially on the theoretical side, the symbol m without the suffix often stands for the rest mass.

zero, that is when the velocity is not too small, Mott's formula (4) reduces to the first bracket of (3). The term in β^2 in (3), which is not given in Mott's expression, clearly represents the "relativity effect" for velocities which are not too great. For small angles of scattering this term becomes $\beta^2 \operatorname{cosec}^4 \theta$, the exchange term of Mott and the $\sec^4 \theta$ term may be neglected, and the expression reduces to the classical non-relativistic formula with $m_0/(1-\beta^2)^{\frac{1}{2}}$ in place of m_0.†

Mott's formula predicts, at large angles of scattering and large velocities, considerably less scattering than that predicted classically, the ratio being one half for all velocities when θ is 45°. Equation (3) predicts even less scattering than Mott's formula; hence both the exchange effect suggested by Mott and the effect of relativity combine to reduce the scattering much below that predicted classically for the interaction of two point charges obeying Coulomb's law of electro-static repulsion.

2. *Experimental Method.*

The great advantage of the expansion method in investigating the present problem lies in the fact that it is the only method that separates the nuclear scattering clearly and definitely from the electron scattering. Williams and Terroux‡ have considered the scattering of fast β-particles by electrons up to angles of about 12°, actually measuring not the angles of scattering, but the ranges of the recoil electrons. In a total track length of about 18 metres, with β varying from 0·60 to 0·97, they obtained about 70 branches with energies from 7500 to 40,000 volts. This number they compared with the values deduced from Bohr's formula,§

$$\mathrm{P}(\mathrm{Q}) = \frac{2\pi e^2 n}{m_0 v^2} \Big/ \mathrm{Q}^2,$$

where P(Q) is the probability of formation of a branch track having an energy between Q and Q + dQ, v being the velocity of the incident particle. In obtaining this formula the electrons were treated classically as particles but the effects of relativity were considered. At higher velocities the observed scattering was about twice that predicted theoretically but the ratio decreased with decreasing velocity.

† It should be observed that we cannot call the formula so obtained, with m (where $m = m_0/(1-\beta^2)^{\frac{1}{2}}$) in place of m_0, the *classical relativistic* formula, for it presumes that the mass of the electron in the atom is equal to that of the incident β-particle.

‡ 'Proc. Roy. Soc.,' A, vol. 126, p. 289 (1930).

§ 'Phil. Mag.,' vol. 30, p. 581 (1915).

The use of an automatic expansion chamber for the production of a large number of photographs of fast β-ray tracks has been described in a previous paper.† The two cameras are arranged with their axes mutually at right angles, and the angles of scattering are computed from measurements of the projected angles on the two films. The velocities of the particles are deduced from the curvatures of the tracks in a magnetic field, the latter being perpendicular to the plane of the chamber.

The present results are from the analysis of 4000 photographs, giving about 30,000 tracks of fast β-particles in nitrogen. Summing the small lengths of track, a total track length of nearly 2 kilometres was obtained which was homogeneous in β to within 10 per cent. In order, however, to measure the curvature of a track with reasonable accuracy for the purpose of deducing the velocity of the particle, it is necessary that the first half of the track should be free from nuclear or electronic deflections. The effective track length available for the actual observation of collisions is thus reduced immediately by one-half. The effective track length for a single particle was estimated at 5 cm. although the total track length visible in the chamber was often about 12 cm. In contrast to previous experiments, the ranges of the ejected electrons were not considered, the actual angle of scattering being measured. The use of relations connecting range and energy was thus avoided.

3. *The Results.*

In 650 metres of track, 250 collisions have been obtained with angles of scattering θ greater than 10°, and with β varying from 0·82 to 0·92. It was not feasible to count values of θ less than 10° owing partly to the comparatively large percentage error which would be introduced and partly to the greatly increased number of measurements it would entail.

For each collision, a dot corresponding to the observed values of θ and β was placed on a diagram with these quantities as co-ordinates, fig. 1. The density of the dots is then a measure of the scattered intensity. The scattering is no longer symmetrical about $\theta = 45°$ owing to the fact that the angle between the two arms of a fork is not now equal to 90° but becomes a function of θ and γ as given by equation (6) in a previous paper.‡ The broken line in fig. 1 indicates the maximum possible angle of scattering for the range of velocities considered here ; it is therefore the line $x = 0$, where x is defined as in equation (1).

† Champion, *loc. cit.*
‡ Champion, *loc. cit.*

2 z 2

The number of collisions is observed to fall off extremely rapidly with increasing values of θ. For the purpose of numerical comparison with theory, another diagram (not shown) was plotted with x and γ as co-ordinates, the integration boundaries being taken from $\gamma = 1\cdot74$ to the end of the β-ray spectrum of radium E at about $\gamma = 3\cdot0$. These limits correspond to a range

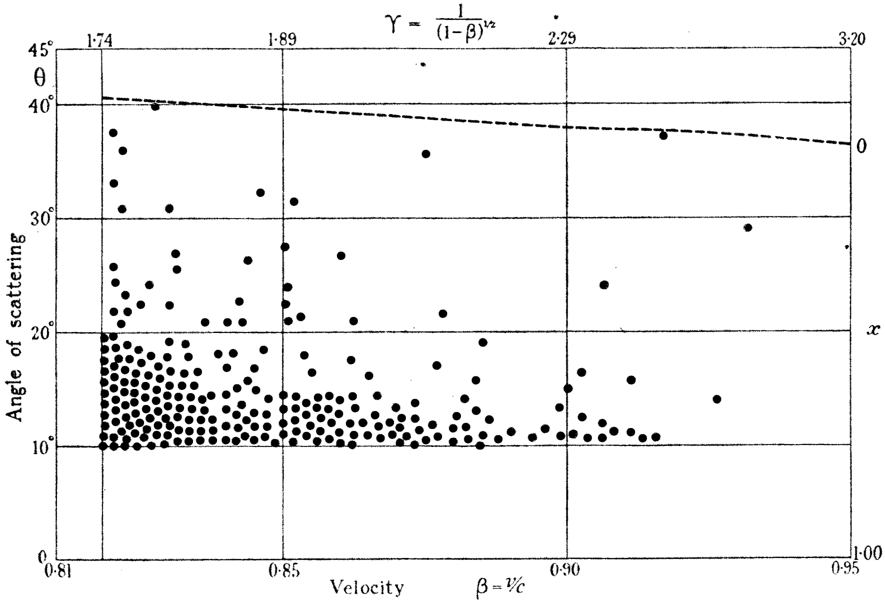

Fig. 1.

in velocity of $\beta = 0\cdot82$–$0\cdot92$, of $H\rho$ from 2400–5000 and of energies from 400,000 to 1,100,000 volts. The number of particles with energies greater than 800,000 volts, however, has been shown in a previous paper† to be only about 5 per cent. of the total number of particles considered. For the integration limits of x those values corresponding to θ equal to 10°, 20°, 30° and maximum, were taken. It may be observed that x, besides being a function of θ, depends also on γ.

The number of particles with velocities lying between values corresponding to γ and $\gamma + d\gamma$, scattered through angles corresponding to x lying between x and $x + dx$, in going a distance dr in the gas is from (2) :—

$$dq = 4\pi \left(\frac{e^2}{m_0 c^2}\right)^2 N_0 f(\gamma, x)\, dx\, dn\, dr, \tag{5}$$

where N_0 is the number of scattering electrons per cubic centimetre of the

† Champion, 'Proc. Roy. Soc.,' A, vol. 134, p. 672 (1932).

gas and dn is the number of β-particles with velocities lying between γ and $\gamma + d\gamma$, while $f(\gamma, x)$ is given by :—

$$f(\gamma, x) = \frac{\gamma^2}{(\gamma^2 - 1)(\gamma - 1)} \cdot \left\{ \frac{4}{(1 - x^2)^2} - \frac{3}{(1 - x^2)} + \frac{(\gamma - 1)^2}{4\gamma^2} \left[1 + \frac{4}{(1 - x^2)} \right] \right\} \quad (6)$$

The quantity dn was obtained from data previously given by the writer.† It is clear that the chief factor influencing the decrease in the number of dots as we pass from left to right across the scattering diagram is the distribution of γ in the β-ray spectrum of radium E and not simply the smaller chance of scattering for a higher value of γ.

The integrations had, of course, to be carried out graphically. The final results are shown in Table I. Column 1 contains the angular limits of θ, and column 2 contains the observed numbers of dots in the three cells. The numbers predicted by Möller's theory are given in the next column, while column 4 contains the values deduced from classical non-relativistic theory. Column 5 contains the values deduced from Mott's equation (4) ; in the next column are given the values obtained by substituting $m_0/(1 - \beta^2)^{\frac{1}{2}}$ for m_0 in Mott's formula. Column 7 gives the values obtained by treating the classical non-relativistic formula in the same way, while the last column gives the values obtained from this formula with T^2 in the denominator in place of $(\frac{1}{2}.m_0v^2)^2$, where T is the initial kinetic energy of the incident β-particle.

Table I.

1.	2.	3.	4.	5.	6.	7.	8.
θ°.	Observed.	Möller.	C.	M (Mott).	M $(1-\beta^2)$.	C $(1-\beta^2)$.	C/T².
30–max.	10	13	57	28	7	15	9
20–30	26	30	148	105	26	37	21
10–20	214	230	761	650	162	190	108
Total	250	273	966	783	195	242	138

4. *Discussion.*

First comparing the total scattering for θ greater than 10°, it will be seen that the observed values and those calculated from Möller's formula are in

† Champion, 'Proc. Roy. Soc.,' A, vol. 134, p. 672 (1932).

good agreement.† The classical non-relativistic formula predicts about $3\frac{1}{2}$ times as much total scattering, but if corrected by writing T^2 in the denominator for $(\frac{1}{2}m_0v^2)^2$, it gives about half the observed value. Writing $m_0/(1-\beta^2)^{\frac{1}{2}}$ for m_0 gives a value of the right order. It is worth noting that this treatment of the classical formula gives also to a first approximation the formula deduced by Mott‡ on quantum principles for the *nuclear* scattering of β-particles. Mott's formula for electron scattering gives about three times as much as that observed and when modified by the above treatment gives too little as shown in the next column.

Examining the distribution of the scattering with varying θ we observe good agreement with Möller's formula. The non-relativistic formulæ are all inapplicable but either $M(1-\beta^2)$ or C/T^2 give good agreement for angles greater than 20°. For angles less than 20° both these corrections give values which are too low, $C(1-\beta^2)$ being more satisfactory. This is to be expected, for we have shown that equation (3) reduces to $C(1-\beta^2)$ for small angles of scattering. The fact that the observed scattering falls so much below that predicted classically for angles greater than 10° indicates that the scattering below this angle must, in some region, be considerably greater than the classical values. This may account for the excess scattering found by Williams and Terroux for the branch tracks up to an angle of scattering of about 10°.

The only other experiment so far performed on the scattering of fast β-particles by electrons is that of Henderson,§ who used the annular ring method. With angles of scattering between 10° and 30°, the results showed that hydrogen and helium possessed a scattering power for fast β-particles considerably in excess of that to be expected from classical considerations of the nuclear and electronic scattering powers of these elements. The conclusion that the observed electronic scattering was about three times that to be expected classically, after a certain correction had been made for the effects of relativity, cannot be said to be in good agreement with the present results, for Henderson's correction was essentially that adopted in column 7 of Table I and the present

† It should be remarked that the experimental difficulties encountered in determining the numbers of dots in the lowest cells of the scattering diagram for the scattering of slow α-particles by helium (Blackett and Champion, 'Proc. Roy. Soc.,' A, vol. 130, p. 380, 1931) are not encountered in the corresponding process in the scattering of fast β-particles by electrons. This is because it is very *difficult* to miss a small angle collision of a fast β-particle with an electron, for the ionisation along the slow branch is much denser than that of the parent track or any general background of cloud that may be present.

‡ 'Proc. Roy. Soc.,' A, vol. 124, p. 425 (1929); *ibid.*, vol. 135, p. 429 (1932).

§ 'Phil. Mag.,' vol. 8, p. 847 (1929).

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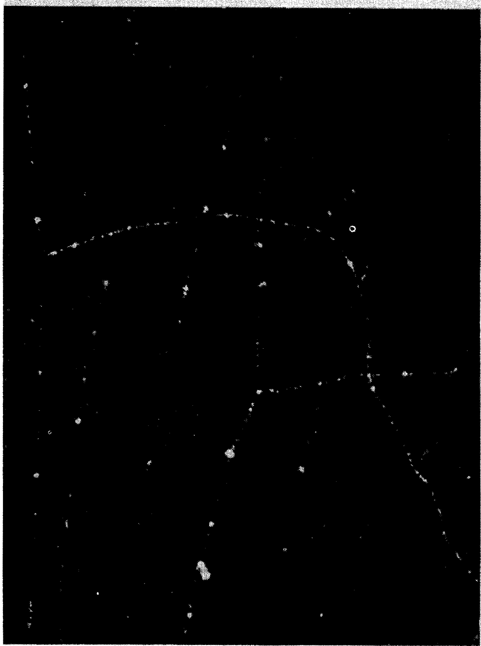

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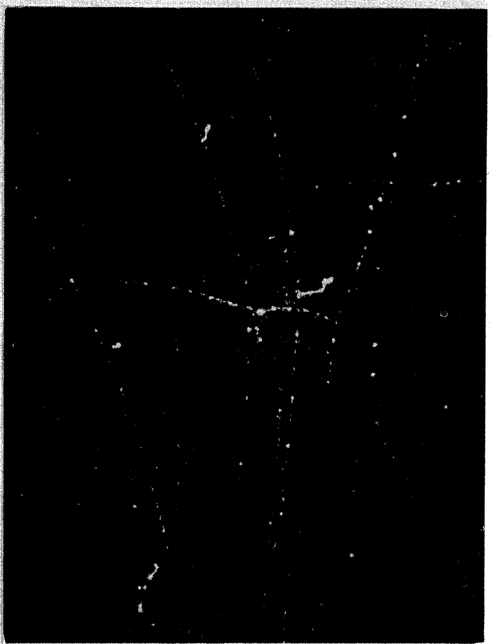

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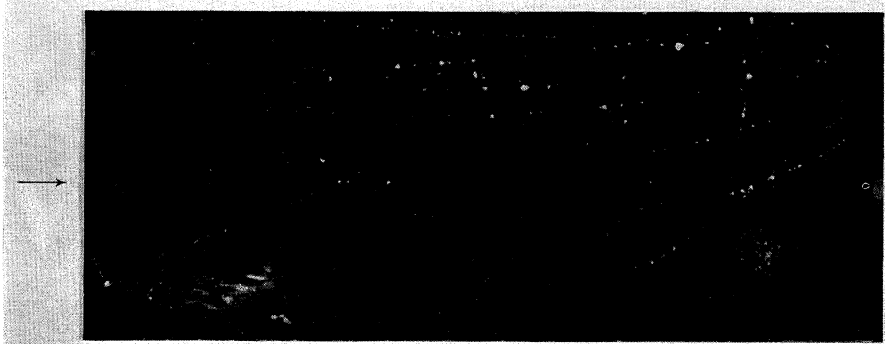

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(*Facing p.* 695.)

results are observed to indicate a total scattering only slightly in excess of that calculated theoretically on this basis. Both experiments, however, show that the observed values are much less than those to be expected if the electron behaved *classically* as a small magnet with a magnetic moment equal to one Bohr magneton. It is now clear that since the de Broglie wave-lengths associated with the β-particles of the energies considered here are about 100 times the closest distance of approach as calculated classically, classical conceptions are inapplicable. It is concluded that Möller's formula gives the best account of the scattering of electrons by electrons.

Summary.

From the analysis of over half a kilometre of track of fast β-particles in nitrogen, photographed by the expansion method, 250 collisions with atomic electrons have been obtained in which the angle of scattering is greater than 10°. The velocities of the incident particles lay between 0·82 and 0·92 that of light.

The absolute numbers scattered and the distribution with angle were in good agreement with a formula of Möller, based on quantum mechanics.

It is a great pleasure to thank Mr. P. M. S. Blackett for much valuable advice and criticism throughout the present work.

PLATE 22.

Description of Photographs (about one-and-a-quarter times natural size).

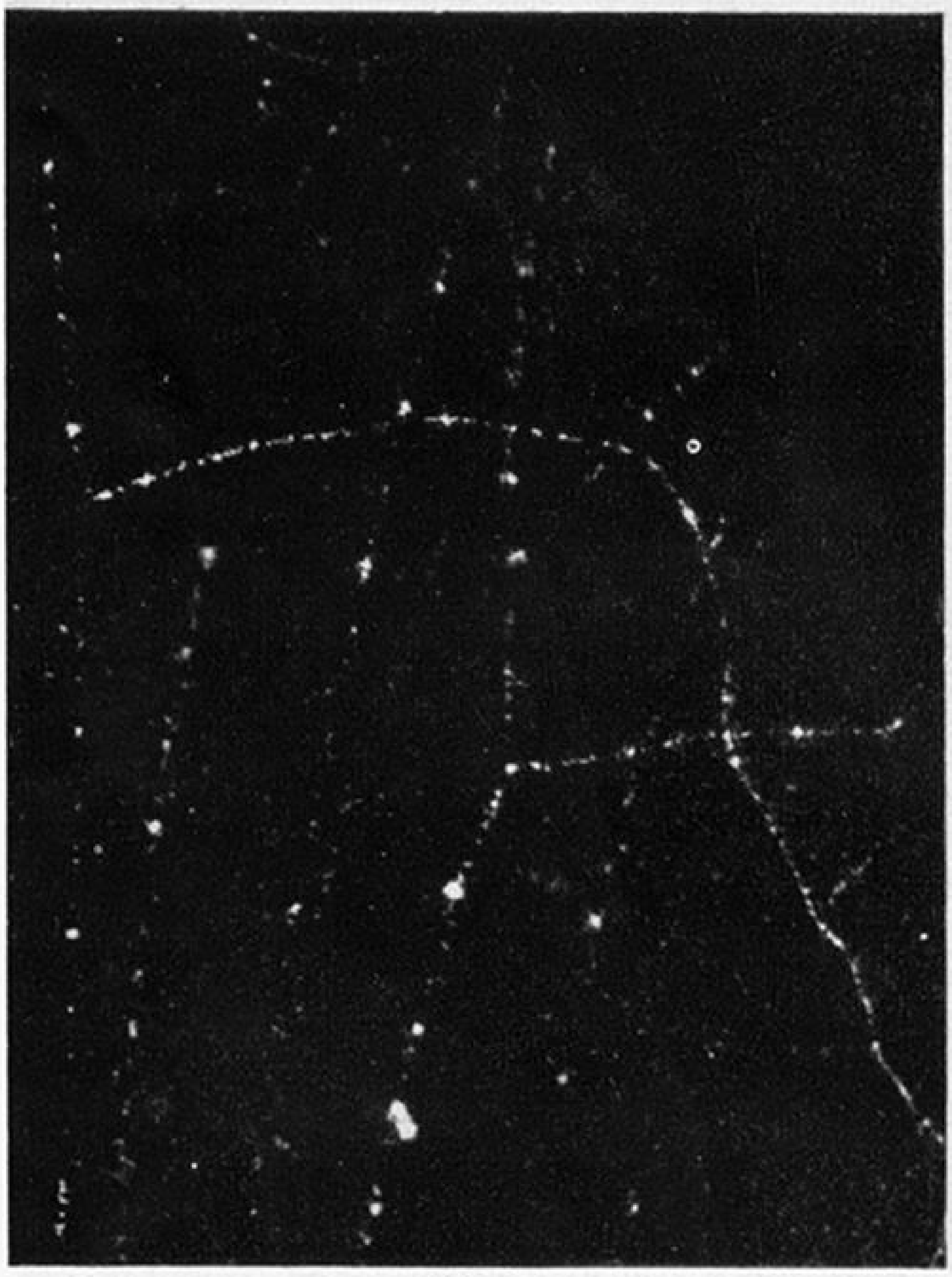
(1) This shows a collision near the lower limit of angle measured, having $\theta = 12°$. The recoil electron is observed to make another collision near the end of its range.

(2) Two close collisions occur here on the same photograph. For the right-hand fork $\beta = 0 \cdot 82$, and it was therefore just included in the measurements. For this fork $\theta = 30°$. The left-hand fork has $\beta = 0 \cdot 90$ and $\theta = 18°$. In this case the collision occurred in the plane of the chamber and consequently the arms of the branch remain in the illuminating beam and also in focus.

(3) Two collisions, one just below the angular limit of measurement and the other having $\beta = 0 \cdot 88$ and $\theta = 14°$.

(4) A close collision with $\beta = 0 \cdot 83$ and $\theta = 31°$.

1



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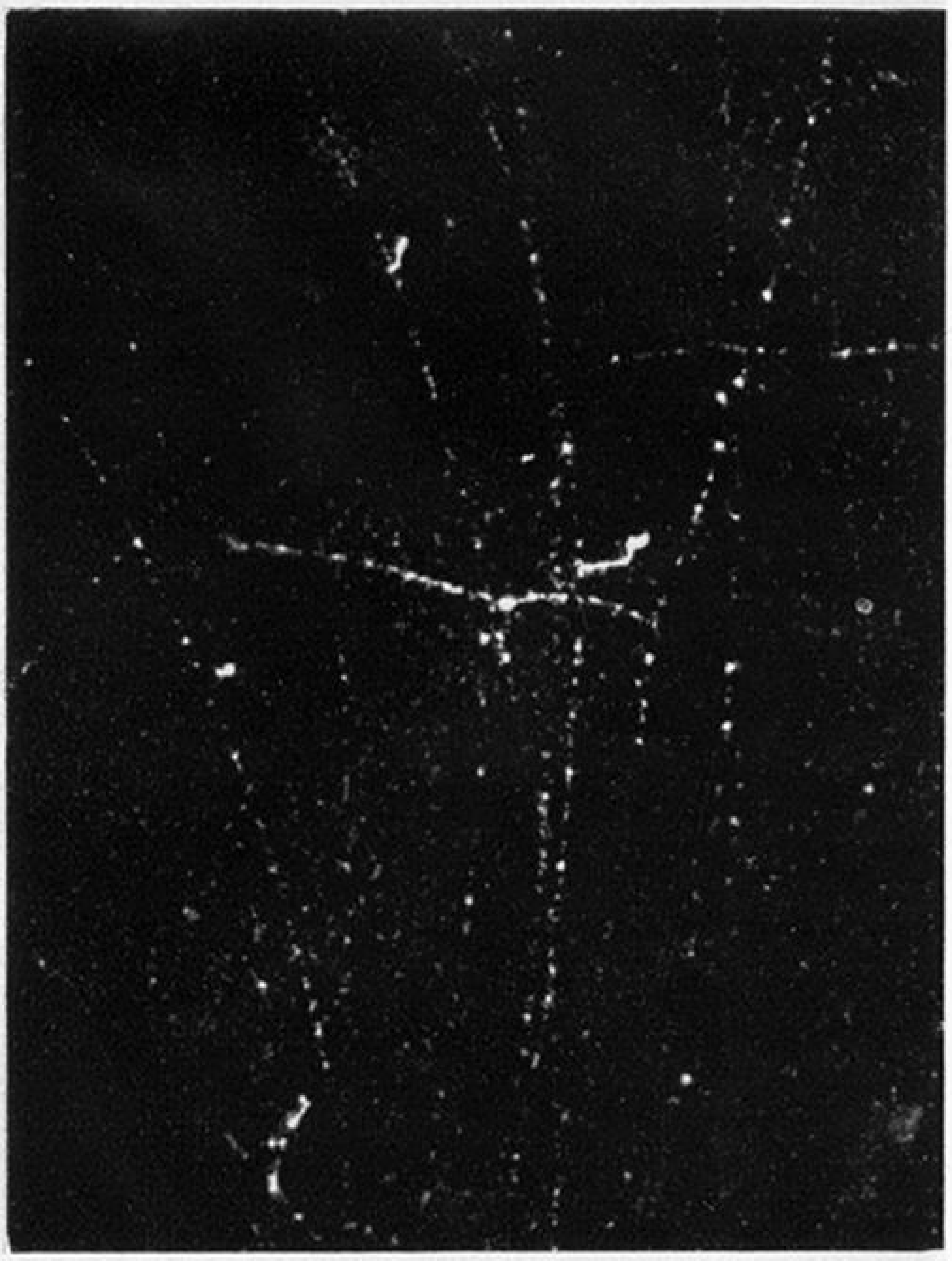

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