transmitted through a mica window into an experimental chamber at atmospheric pressure where their ranges are measured.

The equipment of the laboratory has been made possible by a special grant from the University.

We wish to express our gratitude to Professor Lord Rutherford for his constant encouragement and support in this work. One of us (E.T.S.W.) is indebted to the Department of Scientific and Industrial Research for a Senior Research award.

On some Close Collisions of Fast β-Particles with Electrons, Photographed by the Expansion Method.

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[Plates 12 and 13.]

1. *Introduction.*

The present paper gives an account of measurements on some close collisions of fast β-particles with electrons, photographed by the Wilson cloud method. These measurements afford a direct test of the applicability of the principles of the conservation of momentum and energy and the principles of relativistic mechanics to individual atomic phenomena.

On the basis of Newtonian mechanics, if one particle collides with another which is initially at rest and the two particles are of equal mass, the angle between the directions of motion of the two particles after collision is equal to 90° for all angles of scattering of the incident particle. On relativistic mechanics, however, this angle becomes a function of the angle of scattering and the velocity of the incident particle, and in particular, it becomes smaller and smaller as this velocity approaches that of light. Qualitative evidence has already been given by Wilson,* Bothe† and others that this angle is less than 90° for the collisions of fast β-particles with electrons, but up to the present no quantitative study has been made of the general relation between

* 'Proc. Roy. Soc.,' A, vol. 104, p. 1 (1923).
† 'Z. Physik,' vol. 12, p. 117 (1922).

the whole angle after collision, the angle of scattering, and the velocity of the incident particle. Assuming that momentum and energy are conserved in the collision the following calculation may be made.

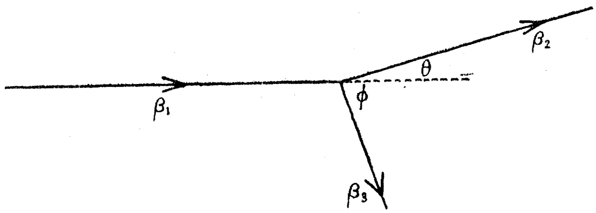


FIG. 1.

Consider the incident electron to have a velocity v_1 with respect to the observer and the stationary electron and let $\beta_1 = v_1/c$. Let the final directions of motion of the two electrons be inclined at θ and ϕ to the initial direction of motion of the incident electron and the velocities in these two directions correspond to β_2 and β_3 respectively. Then from the relativistic momentum and energy relations :—

$$\beta_1\gamma_1 = \beta_2\gamma_2 \cos\theta + \beta_3\gamma_3 \cos\phi \tag{1}$$

$$\beta_2\gamma_2 \sin\theta = \beta_3\gamma_3 \sin\phi \tag{2}$$

$$\gamma_1 + 1 = \gamma_2 + \gamma_3, \tag{3}$$

where

$$\gamma = \frac{1}{(1-\beta^2)^{\frac{1}{2}}}.$$

From these we obtain

$$\beta_2 = \frac{\beta_1 \sin\phi}{\{\beta_1{}^2 \sin^2\phi + \sin^2(\phi+\theta)/\gamma_1{}^2\}^{\frac{1}{2}}} \tag{4}$$

$$\beta_3 = \frac{\beta_1 \sin\theta}{\{\beta_1{}^2 \sin^2\theta + \sin^2(\phi+\theta)/\gamma_1{}^2\}^{\frac{1}{2}}}, \tag{5}$$

and*

$$\cos(\phi+\theta) = \frac{(\gamma_1 - 1)\sin\theta\cos\theta}{\{(\gamma_1+1)^2\sin^2\theta + 4\cos^2\theta\}^{\frac{1}{2}}}. \tag{6}$$

For the symmetrical case in which $\phi = \theta$ and $\beta_2 = \beta_3$, (7) reduces to the simple form

$$\cos\theta = \frac{\beta_1}{(2/\gamma_1 + 3\beta_1{}^2 - 2)^{\frac{1}{2}}}, \tag{7}$$

a result obtained by Bothe.†

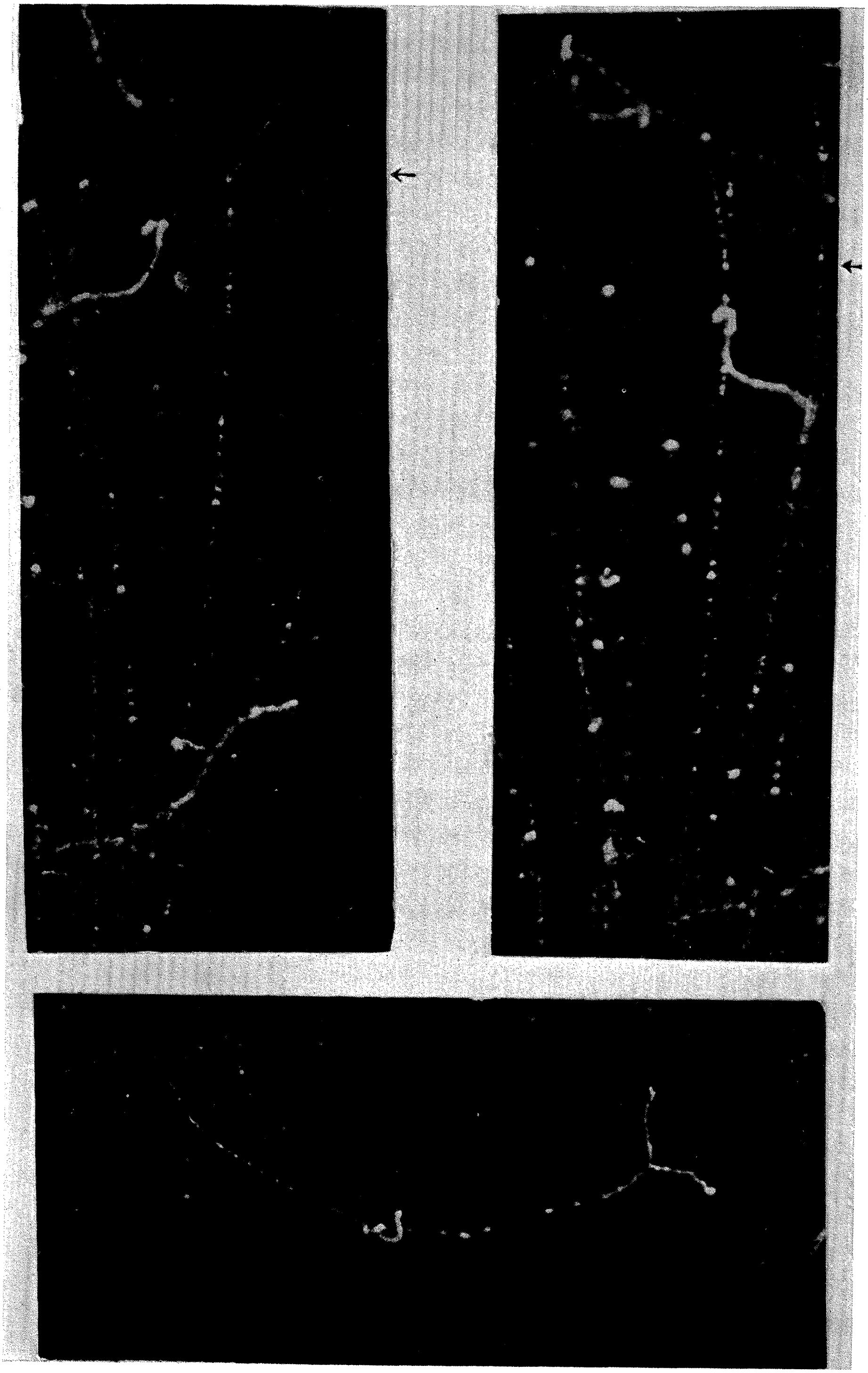
* Wolfe, 'Phys. Rev.,' vol. 137, p. 591 (1931).
† *Loc. cit.*

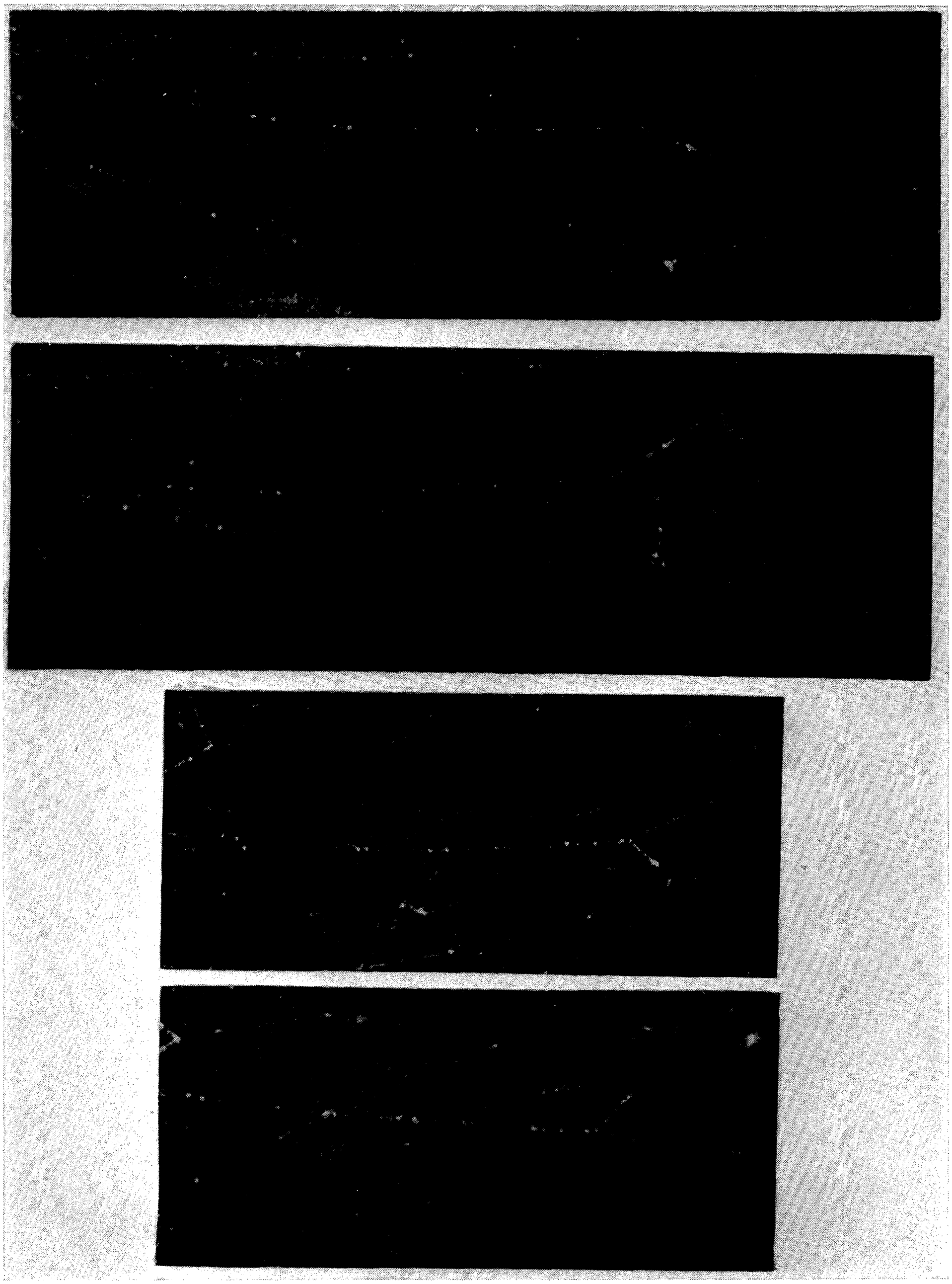
2. *Experimental Method.*

The experimental arrangement of the automatic expansion chamber has been described in a previous paper* ; it utilises the disposition of two cameras with their axes mutually at right angles. The application of the expansion chamber method to the present problem becomes possible only when there is available a large number of photographs of fast β-ray tracks, for the chance of a close collision with an atomic electron is certainly very small. A fork which is being used for the investigation must obey the following conditions for the value of $(\theta + \phi)$ to depart most from 90°. The collision must be as nearly as possible a symmetrical one. This type of collision is very infrequent. Again, the velocity of the incident electron must be as large as possible. The source of particles used in the present experiment was radium E ; the velocities of these particles range up to about $\beta_1 = 0 \cdot 94$, but the fraction with β_1 greater than $0 \cdot 90$ is only a few per cent. of the total number of particles emitted. Even with a source of considerable strength, therefore, a large number of photographs is required before one of these shows a very high speed particle making a close collision.

The velocities of the incident particles were determined from the curvatures of the tracks in a magnetic field which was perpendicular to the plane of the chamber, Champion, *loc. cit.* Artificial tracks of known curvature were placed in the position of the natural tracks in the plane of the chamber and the photographs so obtained were fitted to those of the natural tracks by process of trial and error. For the maximum accuracy in the determination of v_1 the curvature should be as large as possible, but this decreases the accuracy of measurement of ϕ and θ, for which purpose the arms of the fork should have as little curvature as possible. Some balance has therefore to be struck concerning the value of the magnetic field. The value used in the earlier experiments was about 180 gauss, but the curvature produced in the fastest tracks was insufficient for accurate work and in all the later experiments this value was raised to about 250 gauss. With the latter field a particle with $\beta_1 = 0 \cdot 90$ has on the relativity calculation a path of radius of curvature $\rho = 14$ cm. For accurate determination of this curvature (say to 2 per cent.) it is necessary to have about 9 cm. of the track of the incident electron free from nuclear bends. This will again be of rare occurrence, and the adherence to this criterion necessitated the rejection of several otherwise technically good collisions. Further, a collision suitable for accurate measurement, should have the individual drops deposited as evenly

* Champion, 'Proc. Roy. Soc.,' A, vol. 134, p. 672 (1932).





1 (*a*)

1 (*b*)

2 (*a*)

2 (*b*)

as possible along the three branches of the fork; sporadic dense clumps of ions separated by blank spaces of no ionisation render the angle determinations less accurate especially when the curvature of the tracks is taken into account. It is very rare indeed that all these criteria are satisfied simultaneously; the collisions quoted quantitatively here satisfy all the more important criteria. The close collision in Plate 12 satisfies nearly all the required conditions, and in addition it occurs almost exactly in the plane of one of the cameras so that the correction for the true spatial angle is very small and the apparent angle on the left-hand photograph is almost the true value of $(\theta + \phi)$.*

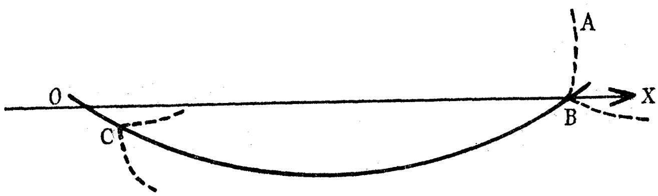


Fig. 2.

4. *Results and Discussion.*

Some 30,000 tracks of fast β-particles in nitrogen and about 5000 in oxygen have been investigated. About 50 close collisions with electrons were observed, and in all cases the measured angles agreed approximately with the values predicted theoretically from relativity considerations. Measurements on 14 of these collisions which satisfy best the more important criteria for accurate quantitative investigation are shown in Table I. In the first column is given the measured value of β_1, the second column giving the corresponding value of the energy of the incident electron in electron volts. In the third column is given the measured value of θ, while the fourth column contains the observed

* Let OX represent the mean direction of the incident β-particle which actually traverses a curved path such as OCB. Then owing to the optical arrangement, the errors involved in the calculation of the true spatial angle of any fork from the angles measured on the photograph become very large if either arm of the fork approaches a direction perpendicular to OX such as at BA. The angle between one of the arms and OX cannot be much less than 45° in the collision of particles of equal mass, and if the collision occurs at B or C where the main stem is itself inclined at a considerable angle to OX, one of the arms of the fork must approach this perpendicular direction. It was necessary to reject all collisions of this type and in actual practice this involved the rejection of several otherwise technically good collisions. The collisions in Table I were all such that these analytical errors were small.

To check the velocities of the particles after collision, that is to determine the values of β_2 and β_3 for comparison with the values calculated from (4) and (5), the curvatures of the

Table I.

No.	β_1.	E.	$\theta \pm 0 \cdot 5°$.	$(\theta + \phi)°$.			$\psi°$.
				Obs.	Calc.	Diff.	
1	0·85 ±1 per cent.	457,100	20·0	83·6 ±1·0	82·7 ±1·0	−1·1	2·6
2	0·83 ±1 per cent.	403,400	26·6	81·2 ±1·0	81·7 ±1·0	+0·5	0·0
3	0·83 ±1 per cent.	403,400	31·4	81·0 ±1·0	81·0 ±1·0	0·0	1·0
4	0·82 ±1 per cent.	380,200	22·0	84·1 ±1·0	83·2 ±1·0	−0·9	0·0
5	0·85 ±1 per cent.	457,100	22·2	82·2 ±1·0	81·7 ±1·0	−0·5	1·0
6	0·83 ±1 per cent.	403,400	22·4	82·1 ±1·0	82·3 ±1·0	+0·2	0·0
7	0·84 ±1 per cent.	428,900	23·4	82·7 ±1·0	81·7 ±1·0	−1·0	0·0
8	0·90 ±2 per cent.	658,500	24·5	79·6 ±1·0	77·4 ±1·5	−2·2	3·7
9	0·88 ±1 per cent.	562,400	35·4	76·8 ±0·5	77·3 ±1·0	+0·5	1·1
10	0·85 ±1 per cent.	457,100	21·1	82·7 ±1·0	81·8 ±1·0	−0·9	0·0
11	0·91 ±2 per cent.	700,000	36·9	75·2 ±0·5	75·2 ±1·5	0·0	1·9
12	0·93 ±2 per cent.	885,000	29·6	72·5 ±0·5	72·6 ±1·5	+0·1	3·1
13	0·85 ±1 per cent.	457,100	21·8	82·4 ±1·0	81·4 ±1·0	−1·0	2·5
14	0·82 ±1 per cent.	380,000	36·9	80·6 ±0·5	81·0 ±1·0	+0·4	1·8
15	0·87 ±2 per cent.	523,000	24·7	83·6 ±1·0	79·6 ±2·0	−4·0	9·7

values of $(\theta + \phi)$ and the values calculated from (6). The last column contains the value of the angle between the plane of the arms of the fork and the main stem. The first two collisions were in oxygen, the remainder in nitrogen.

The agreement between the theoretical and the observed values of $(\theta + \phi)$ is seen to be excellent.* The maximum departure of $(\theta + \phi)$ from a right angle occurs in collision 12 where the deviation is as much as 17·5° ; in this instance the mass of the incident electron is nearly three times its rest mass.

arms of the fork must be measured. It is clearly impossible to use the same method as that used for determining the curvature of the main stem unless the collision occurs in the plane of the chamber. Further, if the arms are inclined at any appreciable angle to the plane of the chamber, they rapidly pass out of focus of the cameras, for it is necessary to use an aperture $f/5 \cdot 6$ to secure adequate intensity of the photographic image, and this aperture gives a depth of focus of only about $\frac{1}{2}$ cm. The arms also pass out of the illuminating beam. There is thus rarely sufficient length of arm to make any accurate quantitative determination of the curvature. Even when the three arms lie approximately in the plane of the chamber it is very rare that none of them exhibits a nuclear bend over the distance necessary for any accurate measurement. However, rough measurements have been made for those collisions where the three arms lie in the plane of the chamber and the values of β_2 and β_3, as might be expected from the agreement of the angle relationships in Table I, were found to agree within experimental error with the values calculated from (4) and (5).

* The concept of the Abraham rigid electron (' Ann. Physik,' vol. 10, p. 105 (1903)) has been applied to the present results for the special case where the energy of the electrons is shared equally after collision. Collision 11 in Table I approximates to this special case and it is found on inserting the Abraham formula for the mass in the momentum and energy equations that $(\theta + \phi)$ is 58° ; actually $(\theta + \phi)$ equals 75·2° in excellent agreement with the value calculated from the relativity expression.

With regard to the errors of measurement, the error in β_1 depends upon the errors in the curvature ρ and the field H. Combining the expression $m = m_0/(1 - \beta_1^2)^{\frac{1}{2}}$ with the expression $mv^2/\rho = \mathrm{H}ev$, we find for the total random error

$$(\Delta\beta_1)^2 = \frac{a^2}{(1-a)^3}\left[\left(\frac{\Delta \mathrm{H}}{\mathrm{H}}\right)^2 + \left(\frac{\Delta\rho}{\rho}\right)^2\right], \tag{8}$$

where

$$a = \left(\frac{m_0 c}{e} \cdot \frac{1}{\mathrm{H}\rho}\right)^2.$$

The error in ρ ranges from 2–4 per cent. over the range of curvatures measured and the error in H is not more than 2 per cent. From (8) the error in the determination of β_1 is therefore not greater than 2 per cent. The determination of the error in the theoretical calculation of $(\theta + \phi)$ from (6) is lengthy, but it may be shown that using the value of $\Delta\beta_1$ from (8) and the value of $\Delta\theta$ as estimated from measurement as $\pm 0 \cdot 5°$, $\Delta(\theta + \phi)$ varies from $\pm 1 \cdot 0°$ to $\pm 1 \cdot 5°$ over the range of β_1 considered.

The validity of the theoretical results has depended entirely on the assumption that there has been no loss of momentum or energy in the form, for example, of electro-magnetic radiation, when the electrons collide. It may be shown that if the energy radiated is comparable with that of the incident electron, the momenta of the radiation and the electron are also comparable. No calculation seems to have been made of the radiation to be expected during the collision of two electrons either on classical or quantum theories. Suppose the energy was radiated as a single quantum in the direction in which the incident electron was travelling immediately before collision. Then consideration of the momentum and energy equations for the symmetrical case shows that with $\beta_1 = 0 \cdot 90$ it would be necessary for the incident electron to lose as much as 30 per cent. of its energy before the angle between the two arms of the fork departed from that calculated from (7) by 2°. Assuming, however, that the radiation could be emitted in any direction, the most favourable test would be the resultant general departure of the arms of the fork from coplanarity. It would be necessary in the most favourable case for the incident electron to lose only about 15 per cent. of its energy for the arms of the fork to depart from coplanarity by 5°. This is about twice the average estimated experimental error and such a deviation would therefore be easily detectable. The foregoing estimate assumes both electrons to share the momentum reaction of the departing radiation. If, however, it was confined to one electron, the least detectable radiation loss would be reduced approximately by one-half.

The last collision in Table I is observed to depart from coplanarity by about 10°. This collision is reproduced in Plate 13.

Another possible explanation of the non-coplanarity of the tracks in this case is that immediately after the collision one of the electrons passed close to a nucleus and underwent a nuclear bend which is not resolved on the photograph. It may be remarked, however, that nuclear bends are usually accompanied by an increase in the number of drops in the neighbourhood of the bend, due probably to the absorption of liberated radiation, and there is no evidence of this in the collision under discussion. Considering the total number of collisions measured, it would appear that if any considerable amount of energy is lost by radiation during close encounters, the number of such inelastic collisions is not greater than a few per cent. of the total number of collisions.

It is of interest to observe that Gray,* using β-particles from radium E has reported the excitation of γ-rays when the β-particles impinge on solid matter. In this case the collision is presumably with the comparatively massive nucleus and Skobelzyn,† using the expansion method, has obtained photographs of inelastic collisions of β-particles with nuclei. Several examples of this type of collision were observed in the present photographs. There is theoretical evidence‡ that a few per cent. of the total number of nuclear collisions are likely to be inelastic. However, the probability of radiation in an electron-electron collision is almost certainly less than in a nuclear collision, for the centre of mass of the system coincides with its electrical centre and there is thus no quantity analogous to a dipole moment. It is conceivable therefore that, in general, the system may be perfectly coupled, all the energy being used to accelerate the initially stationary electron.

Summary.

(1) An automatic expansion chamber was used to investigate the close collisions of fast β-particles with electrons.

(2) In all the cases examined, except one, momentum and energy were found to be conserved and the relations predicted by the restricted principle of relativity, found to hold accurately.

* 'Proc. Roy. Soc.,' A, vol. 85, p. 131 (1911); 'Proc. Roy. Soc.,' A, vol. 86, p. 513 (1912).

† 'Z. Physik,' vol. 43, p. 354 (1927).

‡ Kramers, 'Phil. Mag.,' vol. 46, p. 836 (1923); Gaunt, 'Phil. Trans.,' vol. 229, p. 163 (1930); Mott, 'Proc. Camb. Phil. Soc.,' vol. 27, p. 255 (1931).

(3) If any considerable amount of energy is lost as radiation during the encounters, it is estimated that it occurs in not more than a few per cent. of the total number of collisions.

The writer wishes to thank Lord Rutherford for his interest in the work and Mr. P. M. S. Blackett for constant encouragement and advice.

DESCRIPTION OF PHOTOGRAPHS.

Plate 12.

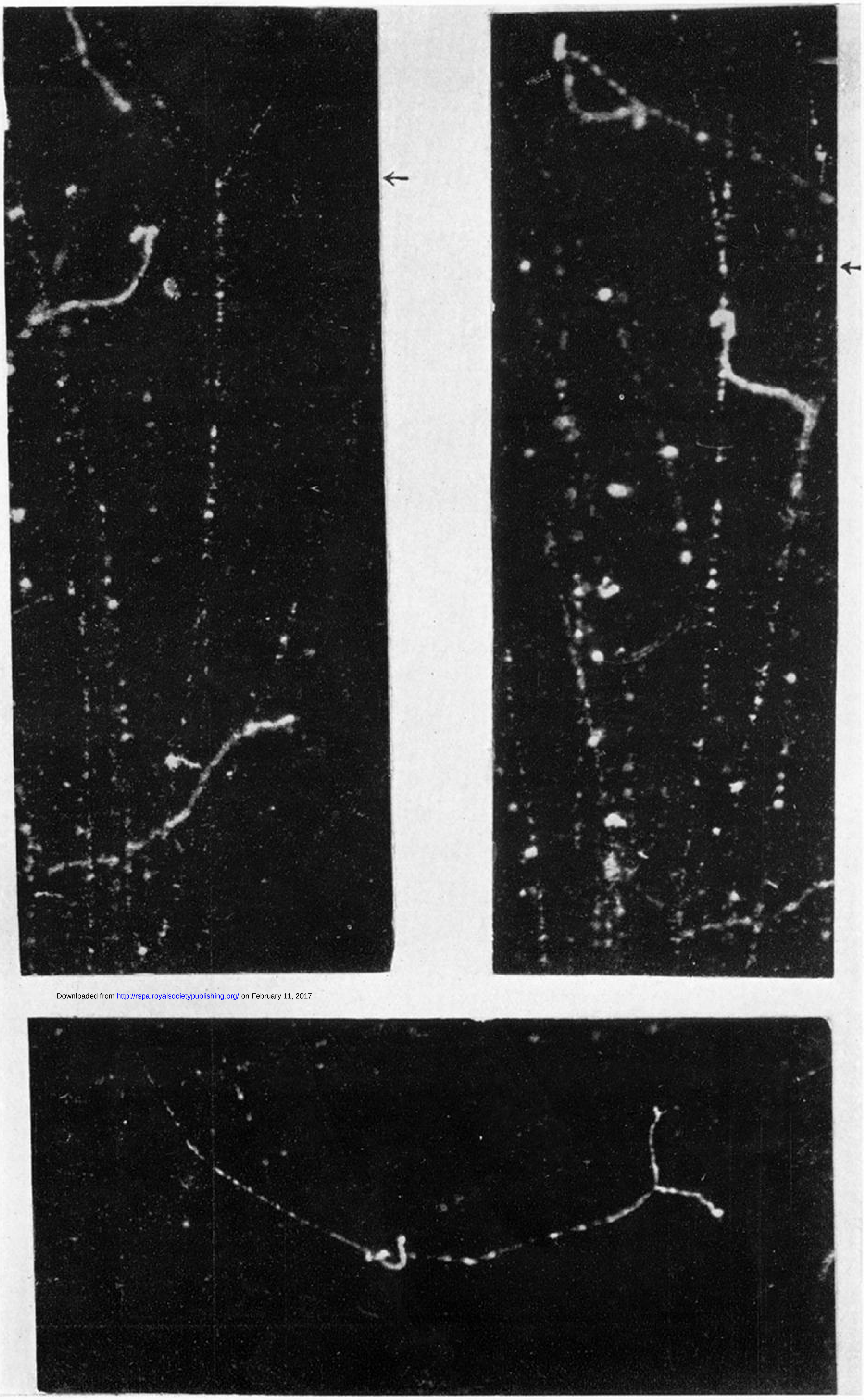
The lower photograph shows a slow electron-electron collision; the angle between the two branches is observed to be about 90°.

The two upper photographs are paired photographs of a collision the measurements of which are given in No. 11, Table I. The angle in the left-hand photograph is almost the actual spatial angle and is observed to be much less than 90°.

Plate 13.

Nos. 1*a* and 1*b* are paired photographs of two collisions which happened to occur on the same photograph, a very rare occurrence. The measurements for the left-hand collision are given in No. 9, Table I. The coplanarity of the three arms of the right-hand collision is strikingly shown in the left-hand photograph.

Nos. 2*a* and 2*b* show the collision the three arms of which are not coplanar.



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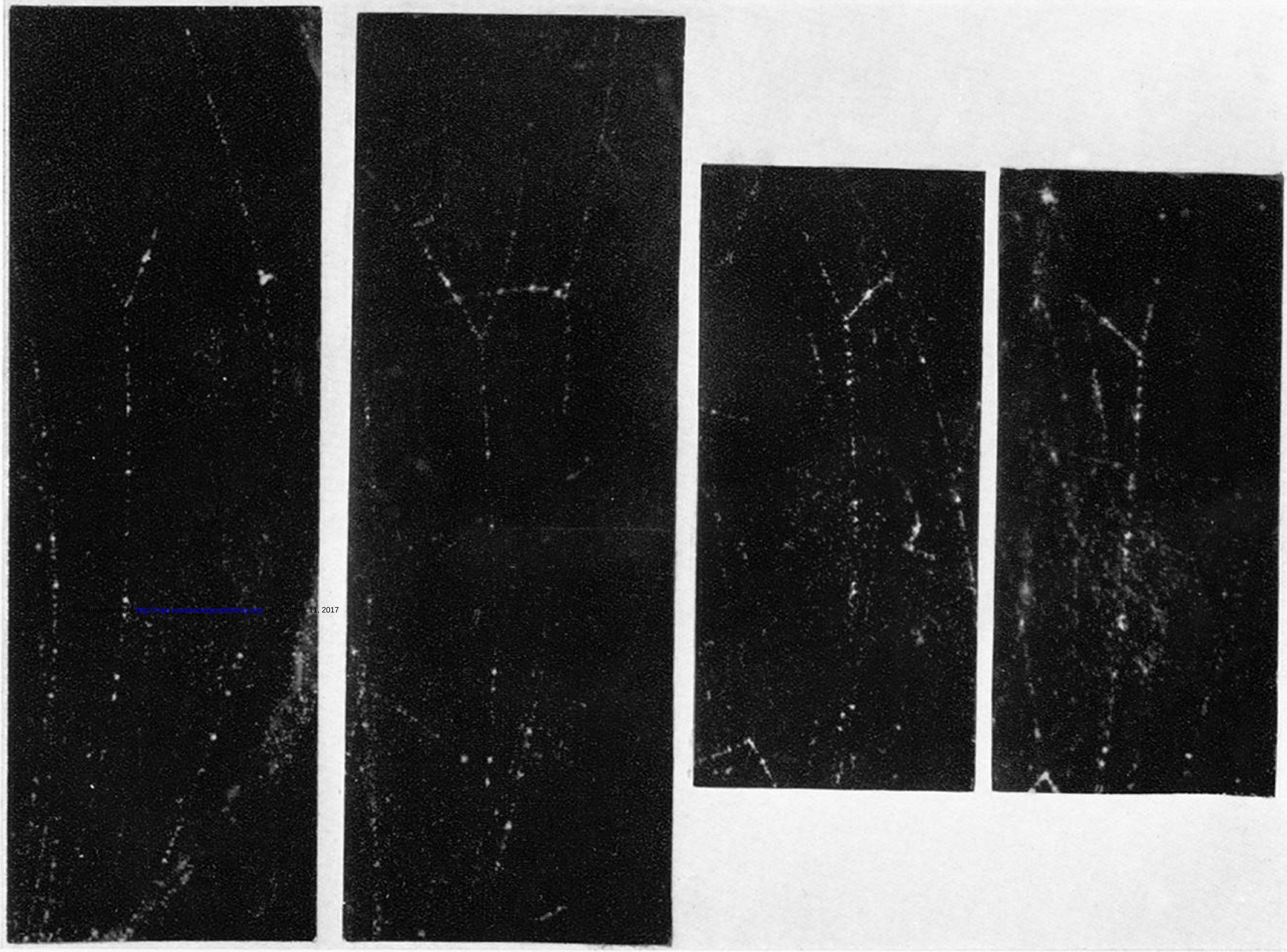

1 (*a*) 1 (*b*) 2 (*a*) 2 (*b*)

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